HARP Memo # xyz Version 1

Fitting HARP data with the Sanford-Wang parametrization of the double-differential, inclusive pion production cross-section

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Abstract

We describe the Sanford-Wang parametrization for the p+A$\rightarrow \pi^+$+X double-differential, inclusive cross-section, and a tentative plan for describing HARP data according to this parametrization, possibly to be used in beam Monte Carlo simulations of neutrino experiments.

1 The Sanford-Wang parametrization

Sanford and Wang first suggested the following empirical formula to describe pion production in proton-nucleus interactions, in the $\sim$10 GeV/c proton momentum range [1]:

$$\frac{d^2\sigma(\mathrm{p+A} \rightarrow \pi^+ + X)}{dpd\Omega}(p,\theta) = c_1 p^{c_2}(1 - \frac{p}{p_{\mathrm{beam}}}) \exp[-c_3 \frac{p^{c_4}}{p_{\mathrm{beam}}^{c_5}} - c_6 \vartheta(p - c_7 p_{\mathrm{beam}} \cos^{c_8} \vartheta)] \quad (1)$$

where X means any other particle in the final state, p_{beam} is the proton beam momentum in GeV/c, p and θ are the π^+ momentum and angle in units of GeV/c and radians, respectively, $d^2\sigma/(dpd\Omega)$ is expressed in units of mb/(GeV/c sr), $d\Omega \equiv 2\pi\ d(\cos\theta)$, and the parameters $c_1, \ldots, c_8$ are obtained from fits to π^+ production data.

The parameter c_1 is an overall normalization factor, the four parameters c_2, c_3, c_4, c_5 can be interpreted as describing the momentum distribution of the secondary pions, and the three parameters c_6, c_7, c_8 as describing the angular distribution for fixed secondary and proton beam momenta, p and p_{beam}. This formula is purely empirical.

This parametrization was first proposed in 1967 to describe pion production data in proton-Beryllium interactions. Since then, results from a number of hadron production experiments in the same energy range have been compared to this parametrization, generally yielding more or less satisfactory fits (see, for example, Ref. [2]). Also, pion production from the interactions of primary protons with the target material in the beam Monte Carlo simulations of neutrino experiments is

often handled according to this parametrization. For example, neutrino flux predictions for both the K2K and MiniBooNE experiments currently use this parametrization for p+Al$\rightarrow \pi^+$+X and p+Be$\rightarrow \pi^+$+X interactions, respectively.

For completeness, we note that, in the literature, an alternative form for Eq. 1 is sometimes used, where the term $(1 - \frac{p}{p_{\rm beam}})$ is substituted with $(1 - \frac{p}{p_{\rm beam}-1})$. The "minus one" at the denominator is introduced to describe threshold effects in pion production. In the following, we ignore this "minus one" and use Eq. 1.

2 Fitting HARP data according to the Sanford-Wang formula

A simple MINUIT χ^2 minimization code has been put together to fit HARP data (when available) according to Eq. 1, by varying the parameters $c_1, \ldots, c_8$. The goals of this program are:

- quantify whether HARP data can be satisfactorily described by Eq. 1, for example by looking at χ^2/d.o.f. for the best-fit χ^2;
- provide the best-fit parameters $c_1, \ldots, c_8$;
- provide the parameter errors and correlations, to be used to estimate the systematic error associated with pion production.

For now, the following simple χ^2 function is assumed:

$$\chi^2(c_i) = \sum_{\alpha}^{N} \frac{(N_\alpha^{\rm data} - N_\alpha^{\rm pred}(c_i))^2}{(\delta N_\alpha^{\rm data})^2} \tag{2}$$

where α labels a HARP (p, θ) bin, N is the number of (p, θ) bins, $N_\alpha^{\rm data}$ is the cross-section measurement for $d^2\sigma/(dpd\Omega)$ in the (p, θ) bin, and $N_\alpha^{\rm pred}$ is the integral of Eq. 1 over the (p, θ) bin widths, divided by the bin widths. The fitting program can be easily modified if different conventions for the pion variables or for the HARP cross-section data will prove to be better, from the experimental point of view.

Equation 2 assumes only uncorrelated errors, at the moment. HARP errors will likely be more complicated than this. An overall normalization systematic error, fully correlated across (p, θ) bins, can be addded by introducing a ninth parameter A in the fit:

$$\chi^2(c_i) = \sum_{\alpha}^{N} \frac{(N_\alpha^{\rm data} - AN_\alpha^{\rm pred}(c_i))^2}{(\delta N_\alpha^{\rm data})^2} + \frac{(A-1)^2}{\sigma_A^2} \tag{3}$$

where σ_A is the normalization systematic error. If other errors, which are not uncorrelated or fully correlated, will be present (HARP momentum scale?), then a more general χ^2 function will be used, for example:

$$\chi^2(c_i) = \sum_{\alpha,\beta}^{N} (N_\alpha^{\rm data} - AN_\alpha^{\rm pred}(c_i))(M^{-1})_{\alpha,\beta}(N_\beta^{\rm data} - AN_\beta^{\rm pred}(c_i)) + \frac{(A-1)^2}{\sigma_A^2} \tag{4}$$

where $M_{\alpha,\beta}$ is an error matrix with non-zero, non-diagonal elements.

The parameter c_3 is assumed to be fixed to the value obtained from fits to previous data. The reason is that Eq. 1 was constructed assuming various datasets with possibly different beam momenta in the fit. Most likely, HARP data at a fixed beam momentum will be used in the K2K and MiniBooNE simulations (12.9 and 8.9 GeV/c, respectively). In this case, keeping all parameters floating in the fit is redundant, so we adopt the convention of fixing c_3, while letting c_5 (and the other parameters) float.

3 Translating HARP into neutrino flux predictions

If HARP data can be satisfactorily fit according to Eq. 1, then a simple procedure to extract central values and systematic uncertainties (from π^+ production data) in neutrino flux predictions can be used.

Central values can be extracted simply by running the neutrino flux Monte Carlo simulation with the parameters $c_1, \ldots, c_8$ that fit best the HARP data, in order to obtain $\phi_\alpha(c_i)$, where α now labels a bin in generated neutrino energy, i labels the eight Sanford-Wang parameters, and ϕ can be a flux prediction at a given detector location, a far/near flux ratio between two detector locations, or something more complicated.

In order to extract a neutrino flux covariance matrix $M^\phi_{\alpha\beta} \equiv \langle \delta\phi_\alpha \delta\phi_\beta \rangle$, seven additional flux Monte Carlo simulations are needed, where each of the Sanford-Wang parameters is individually varied by some amount ϵ_i with respect to its best-fit value (eight simulations and not seven, if c_3 is not fixed). The error matrix is then obtained by standard error propagation:

$$M^\phi_{\alpha\beta} = \sum_{i,j=1}^{N_c} D_{\alpha,i} M^\pi_{i,j} D^T_{\beta,j} \tag{5}$$

where $N_c = 7$ or 8, $M^\pi_{i,j} \equiv \rho_{i,j} \delta c_i \delta c_j$ is the covariance matrix in the Sanford-Wang parameters obtained from the HARP fit, and the derivatives $D_{\alpha,i}$ are:

$$D_{\alpha,i} \equiv \frac{\phi_\alpha(c_{k\neq i}, c_i + \epsilon_i) - \phi_\alpha(c_i)}{\epsilon_i} \tag{6}$$

where, in $\phi_\alpha(c_{k\neq i}, c_i + \epsilon_i)$, only the single parameter c_i is varied by some amount, while the other parameters c_k are specified according to their best-fit values, as in $\phi_\alpha(c_i)$. The amounts ϵ_i should be small, but making sure that the values $D_{\alpha,i}$ are not dominated by Monte Carlo statistical errors. For example, the choice $\epsilon_i = \delta c_i$ could be used. This procedure is very simple, and sophistications might be needed, for example to account for non-linearities in the parameters' variations, or for asymmetric errors. On the other hand, the procedure, as it is, is sufficient to extract a full error matrix in the neutrino flux predictions, including correlations among neutrino energy bins.

References

[1] J. R. Sanford and C. L. Wang, Brtookhaven National Laboratory, AGS internal report, 1967 (unpublished); C. L. Wang, Phys. Rev. Letters **25**, 1068 (1970); *ibid.* **25**, 1536(E) (1970).

[2] Y. Cho *et al.*, Phys. Rev. D **4**, 1967 (1971).